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## 13. ABSTRACT (Maximum 200 words)

The objective of research is to investigate the sensitivity of the frequency-shifts of the guided EM waves in dielectric resonators when they are subjected to external forces, shocks, or vibrations.

Forces applied to a resonator produce stresses which, through the nonlinear piezooptical effect, changes the permittivity constant into an inhomogeneous, second-order permittivity tensor, depending upon the spatial distribution of stresses. This changing in material property, in turn, affects the velocities and couplings of EM waves inside the resonator and hence causes the shifts of resonances.

Closed form solutions of the 3-D Maxwell's equations for finite resonators are very few and they are known only for solid sphere and torus of homogeneous isotropic dielectrics.

In order to study the problems of stress effect on the vibrations in dielectric plate resonators, circular or rectangular, in a systematic way, the research is conducted according to the following sequence.

A system of 2-D equations for guided waves is deduced from the 3-D Maxwell's equations. Closed form solutions are obtained from these 2-D equations for circular disks embedded in free space. Then the stress sensitivity of guided waves in circular isotropic disks and in infinite anisotropic plates are studied.

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# THE STRESS SENSITIVITY OF DIELECTRIC RESONATORS

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## I. STATEMENT OF THE PROBLEM STUDIED

The objective of the research is to investigate the sensitivity of the frequency-shifts of the dielectric resonators when they are subjected to external forces, shocks, or vibrations.

In a resonator, applied forces produce stresses which, through the nonlinear piezooptical effect, changes the permittivity constant into an inhomogeneous, second-order permittivity tensor, depending upon the spatial distribution of stresses. This changing in material property, in turn, affects the velocity and couplings of EM waves inside the resonator and hence causes the shifts of resonances.

The main motivation of the project is its result in engineering application. It is anticipated that the result of the research will reveal the ways to minimize the frequency-shift by controlling various resonator parameters to improve the design for stable dielectric resonators against stress bias caused by external forces, shocks, or vibrations.

Closed form solutions of the 3-D Maxwell's equations for finite resonators are very few and they are known only for solid sphere and torus<sup>1</sup>. For thin circular disk or rectangular parallelepiped of dielectric, the most commonly employed resonators in practical applications, no closed form exact solutions are available at present time.

In order to study the problems of stress effect on the vibrations in dielectric plate resonators in a systematic way, the research is conducted according to the following sequence.

A system of 2-D equations for guided waves is deduced from the 3-D Maxwell's equations. Closed form solutions are obtained from these 2-D equations for circular disks embedded in free space. Then the stress sensitivity of guided waves in circular isotropic disks is studied by a perturbation method.

Exact closed form solutions for guided EM waves in infinite anisotropic dielectric plates with general crystal symmetry are obtained. Based on these solutions, stress effect on the frequencies of guided EM waves as a function of wave number is computed for homogeneous stresses.

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## II. SUMMARY OF THE MOST IMPORTANT RESULTS

1. A variational principle for the equations of piezoelectromagnetism is obtained which includes the variational principles for the equations of elasticity<sup>2</sup>, Maxwell's equations<sup>3</sup>, and equations of piezoelectricity<sup>4</sup> as special cases (See III<sub>1</sub>).
2. A system of 2-D equations for guided waves in dielectric plates is deduced from the 3-D Maxwell's equations. The accuracy of these 2-D equations is established by showing the close agreement of the dispersion relations obtained from the solutions of the 2-D equations with those from the solutions of the 3-D equations (See III<sub>2</sub>).
3. Closed form solutions of the 2-D governing equations are obtained for the vibrations of circular disk dielectric resonators embedded in free space. Predicted results are compared with the experimental data by Cohn<sup>5</sup>, Chow<sup>6</sup>, and Yee<sup>7</sup>, and with computational results by Chow<sup>6</sup> and Yee<sup>7</sup>. It may be seen that present predictions agree well overall with different sets of experimental data and for modes of various types and order (See III<sub>3</sub>).
4. Stress Sensitivity of Resonances in Circular Disk Dielectric Resonators.

Two-dimensional equations for guided EM waves obtained in III<sub>2</sub> are extended to include piezooptical effect in the constitutive equations. These equations and the solutions obtained in III<sub>3</sub> are then used to calculate frequency shift of TE modes caused by applied stresses in circular disk dielectric resonators. A perturbation method is employed by which predictions of the first order frequency shift due to strain or stress are obtained. Work is being conducted in collaboration with Dr. A. Ballato of ETDL (See III<sub>4</sub>).

5. Guided EM Waves in Infinite Dielectric Plates with General Crystal Symmetry<sup>4</sup>.

Guided waves in an infinite dielectric plate with general crystal symmetry surrounded by free spaces are studied in terms of the three-dimensional Maxwell's equations.

To exhibit as how the crystal symmetry may affect the propagation and coupling of the waves, the study is divided into four cases: (I)  $\beta_{11}, \beta_{22}, \beta_{33} \neq 0$ ; (II)  $\beta_{11}, \beta_{22}, \beta_{33}, \beta_{12} \neq 0$ ; (III)  $\beta_{11}, \beta_{22}, \beta_{33}, \beta_{23} \neq 0$ ; (IV) all  $\beta_{ij}$  is the impermeability tensor referred to the rectangular axes  $x_i$  with the  $x_2$  axis normal to the plate faces. For each case, the closed form solutions are obtained, and dispersion relations and modes are computed and studied. These solutions are very much needed in cases where the plates are made of crystal materials or are originally isotropic, but rendered anisotropic either by mechanical stress through the piezooptical effect or by electric field through the electro-optical effect (See III<sub>5</sub>).

6. Effect of Stress on Guided EM Waves in Anisotropic Dielectric Plates.

The effect of uniformly applied normal and shearing stresses on the changes in the velocity or frequency of guided EM waves is being studied for infinite dielectric plates of general crystal symmetry, based on the recently obtained exact solutions (given in III5) of the three-dimensional Maxwell's equations. This work is being conducted in collaboration with Dr. A. Ballato of ETDL (See III6).

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### IV. LIST OF PARTICIPATING SCIENTIFIC PERSONNEL

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Dr. A. Ballato, Principal Scientist, ARL

Mr. J.S. Yang (expecting to obtain his Ph.D. degree within the next 3 months).

Mr. J.D. Yu